EE327 Digital Signal Processing Introduction to Digital Filters

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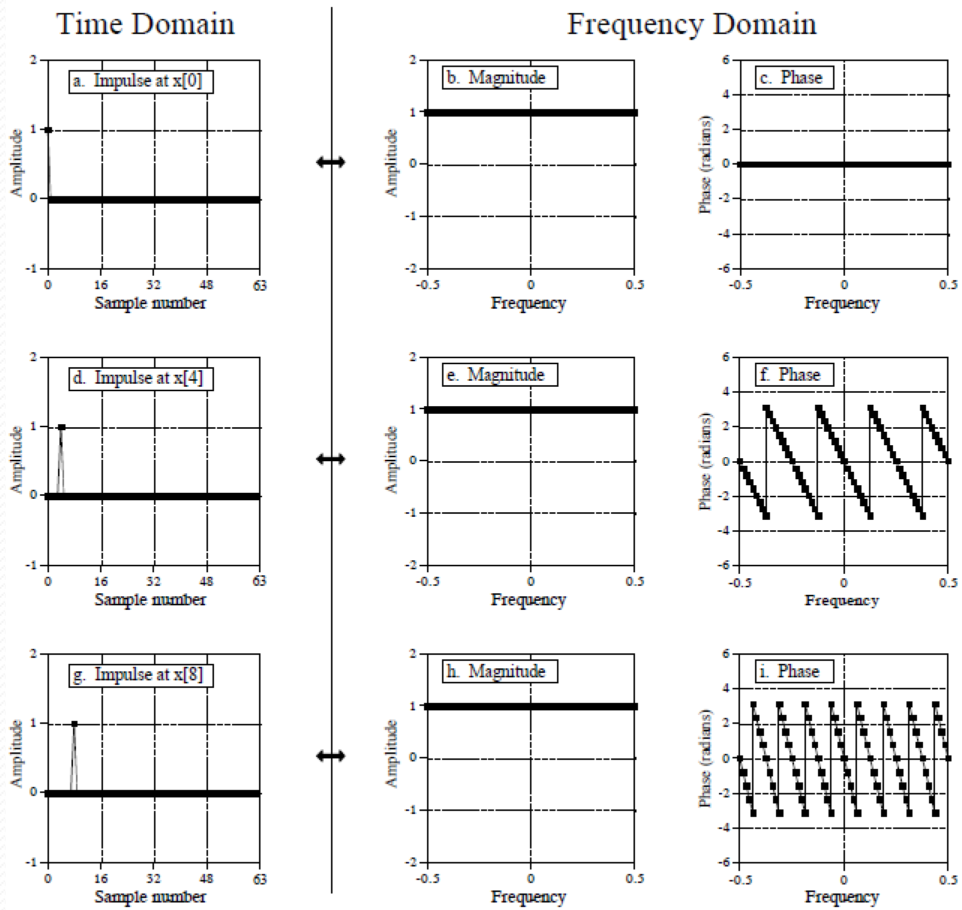
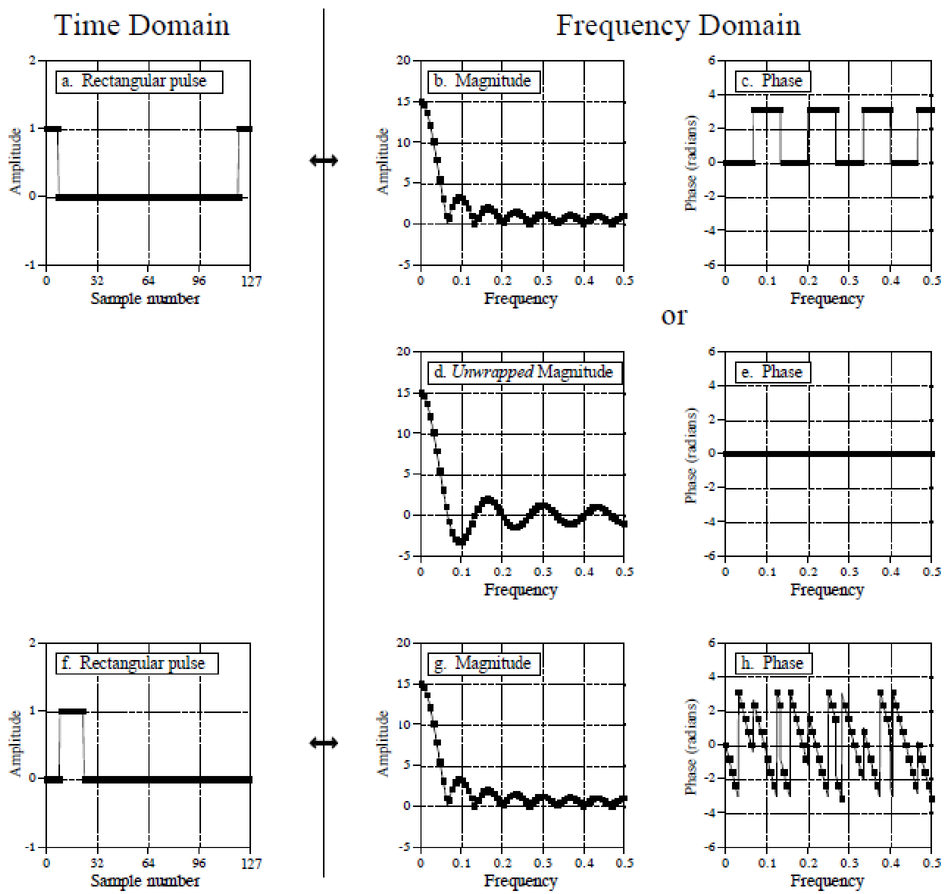
REMINDER 1:
Impulse ←→Constant Magnitude



REMINDER 2: Rectangular Pulse ←→Sinc



$$Mag\ X[k] = \left| \frac{\sin(\pi k M / N)}{\sin(\pi k / N)} \right|$$

REMINDER 3:
Sinc ←→Rectangular Pulse

- Rectangular pulse in frequency domain

↓

$$x[i] = \frac{1}{N}\,\frac{\sin(2\pi i(M - 1/2)/N)}{\sin(\pi i/N)}$$

Fast Fourier Transform

- A fast way to calculate DFT
- FFT: N points signal (Time Domain)
  →N complex numbers (Frequency Domain)
- The last N/2 points are a mirror image of the first N/2
- Only N/2+1 independent real and N/2+1 independent imaginary numbers.
- To work the input MUST be padded with zero to a power of 2 number of points.

Why Filters

- Signal Separation
  - Separate mixed signals
- Signal Restoration
  - Remove the effect of unwanted system on the signal

Why Digital Filters

- Can achieve far more superior results compared to analog filters

How to Represent a Filter

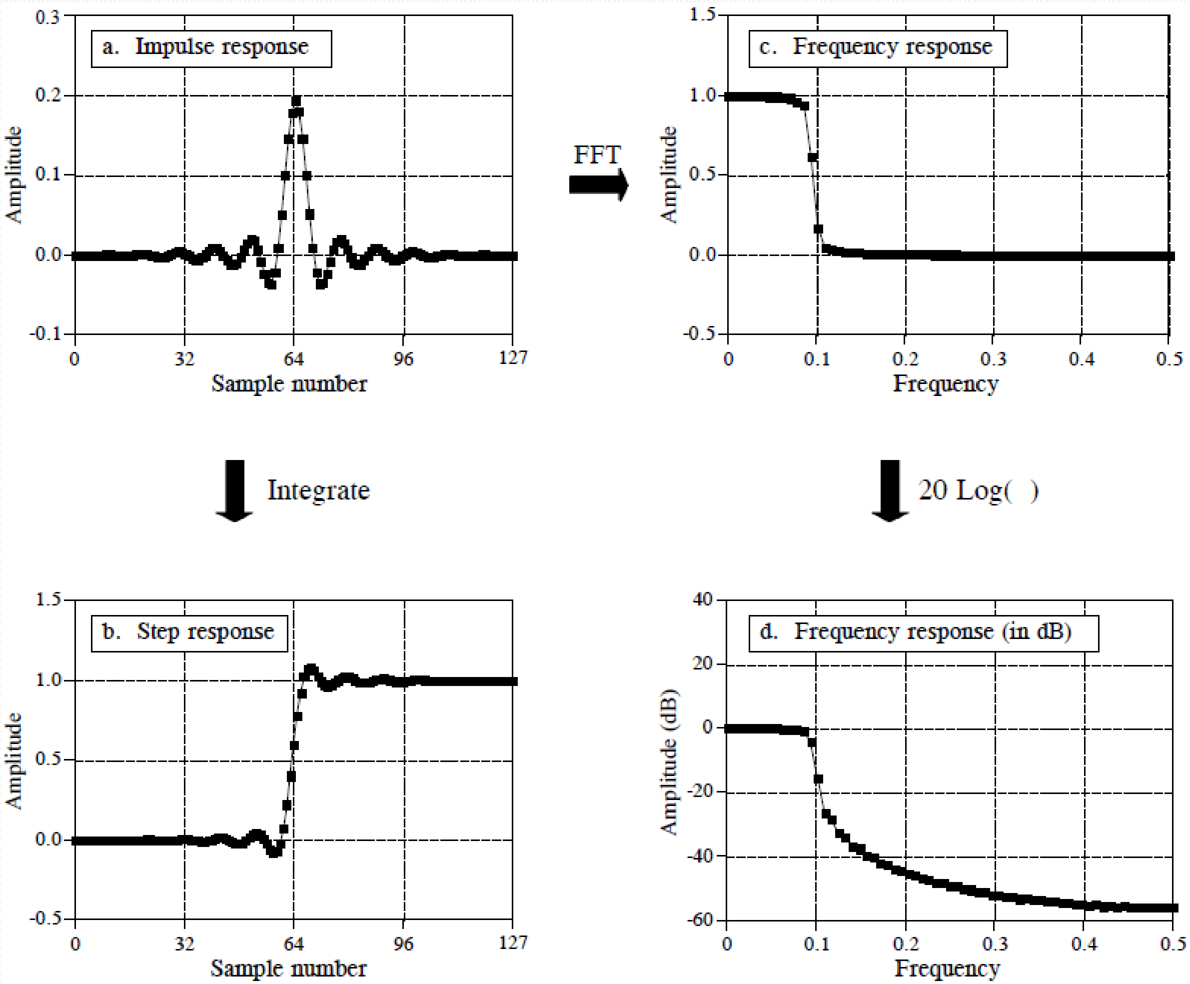
- Finite Impulse Response (FIR)
  - Impulse Response
    - Filter Kernel
  - Step Response
  - Frequency Response

$$y[n] = \sum_{i=0}^{M} a_{-i} x[n-i]$$

- Infinite Impulse Response (IIR)
  - Recursion Coefficient

$$y[n] = \sum_{j=0}^{M_1} a_{-j} x[n-j] + \sum_{i=0}^{M_2} b_{-i} y[n-i]$$

Relation between Kernel Representations



What is a dB

- A bel = an increase in POWER by an order of magnitude
- A decibel = an increase in power by a factor of ONE

$$dB = 10\log_{10}\frac{P_2}{P_1}$$

$$dB = 20\log_{10}\frac{A_2}{A_1}$$

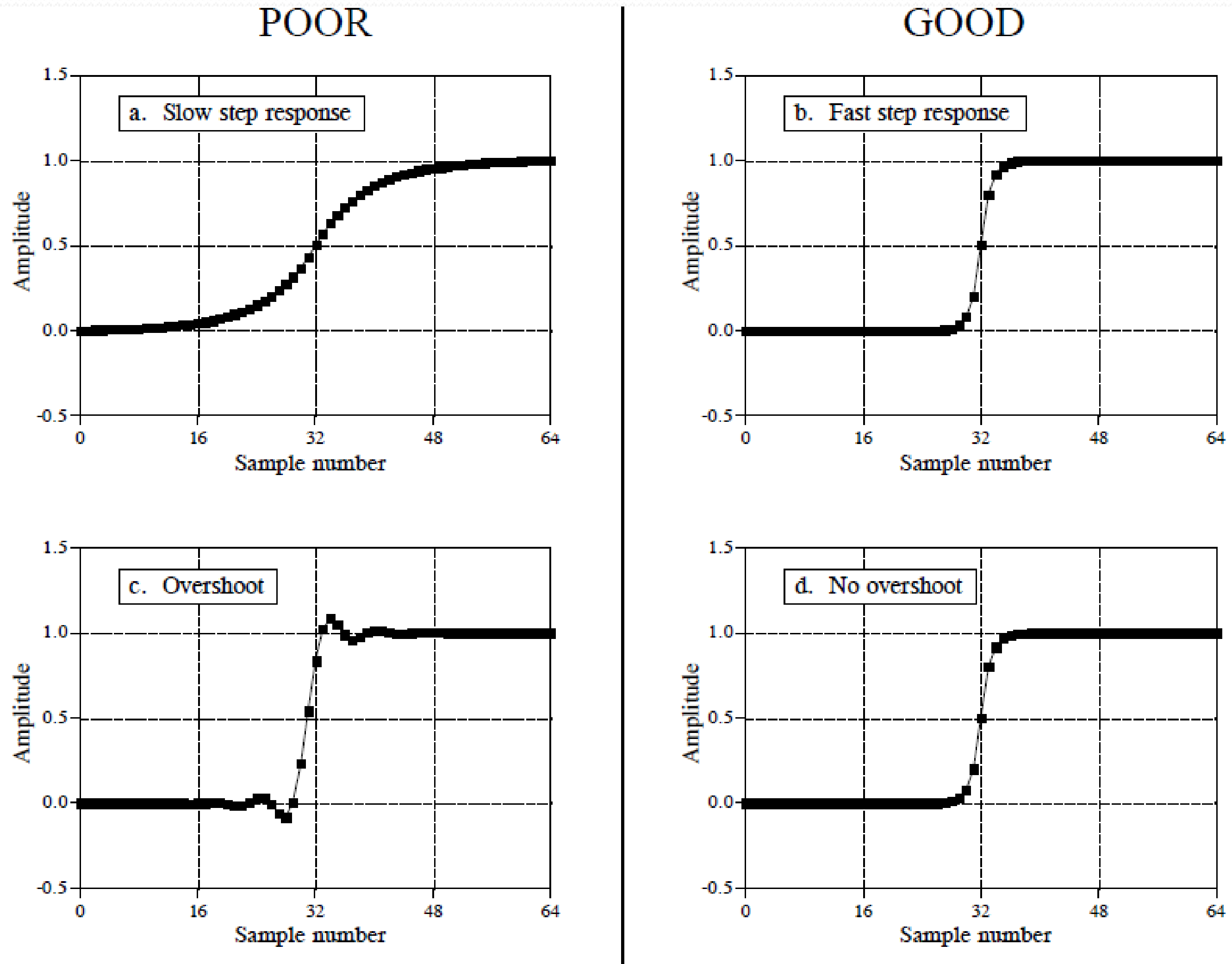
Information Representation

- In time domain
  - Shape
  - E.G. measurement of sun light every second
- In frequency domain
  - Frequency
  - Phase
  - E.G. measurement of a distance between a planet and a star over time.
- Mixed
  - EKG with white noise

Time Domain Parameters

- Step Response
  - Measured by rise time (time to go from 10% to 90% amplitude level)
- Overshooting
- Phase Linearity

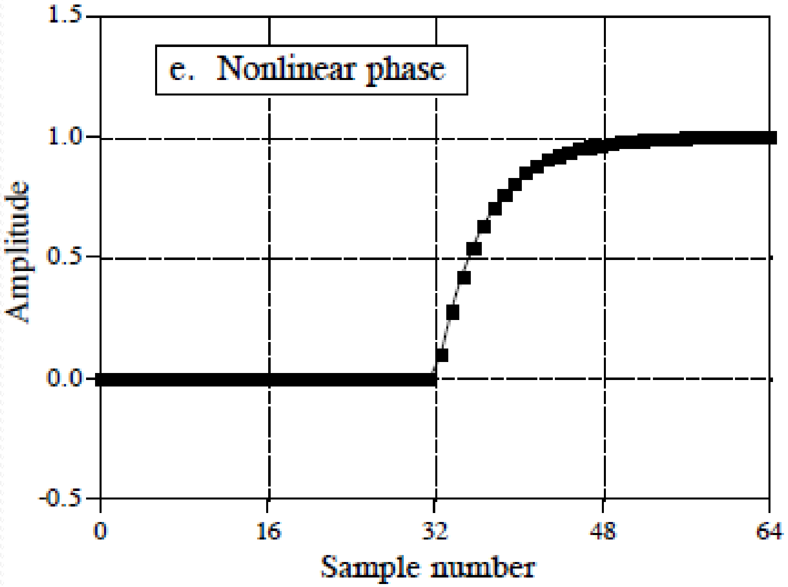
Time Domain Parameters

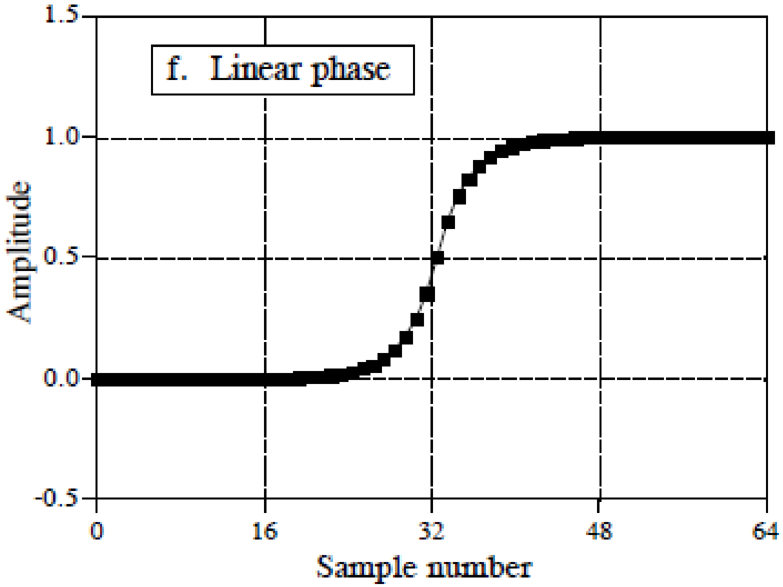


Time Domain Parameters

POOR

GOOD

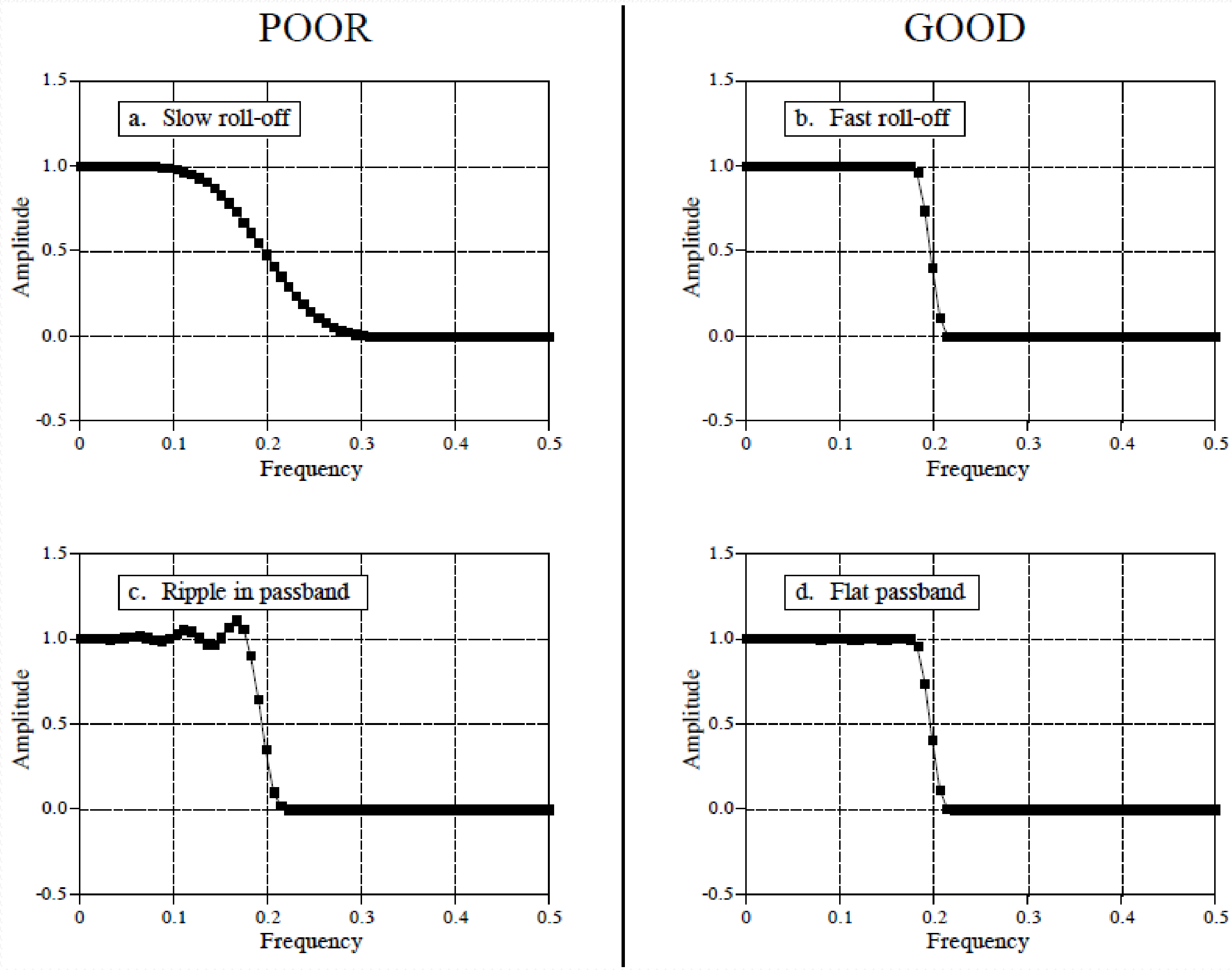




Frequency Domain Parameters

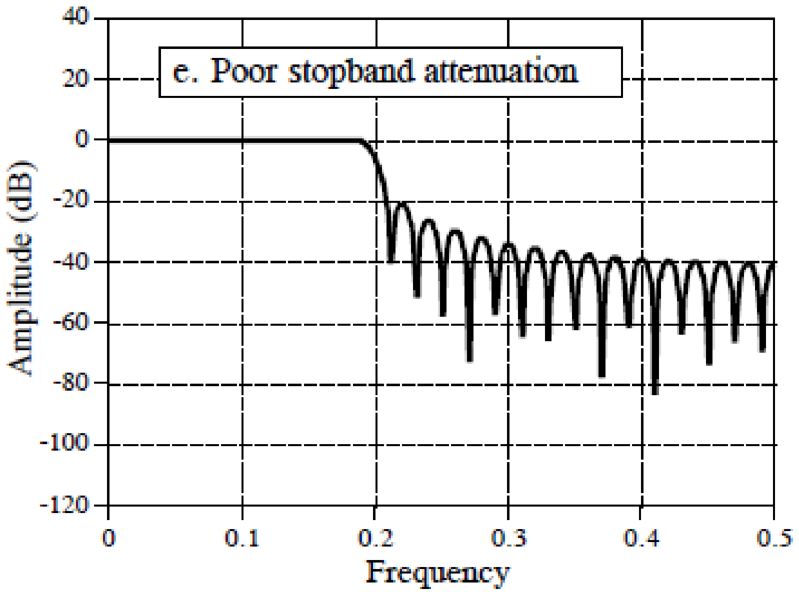
- Roll off
- Ripple in passband
- Stopband attenuation

Frequency Domain Parameters



Frequency Domain Parameters

POOR



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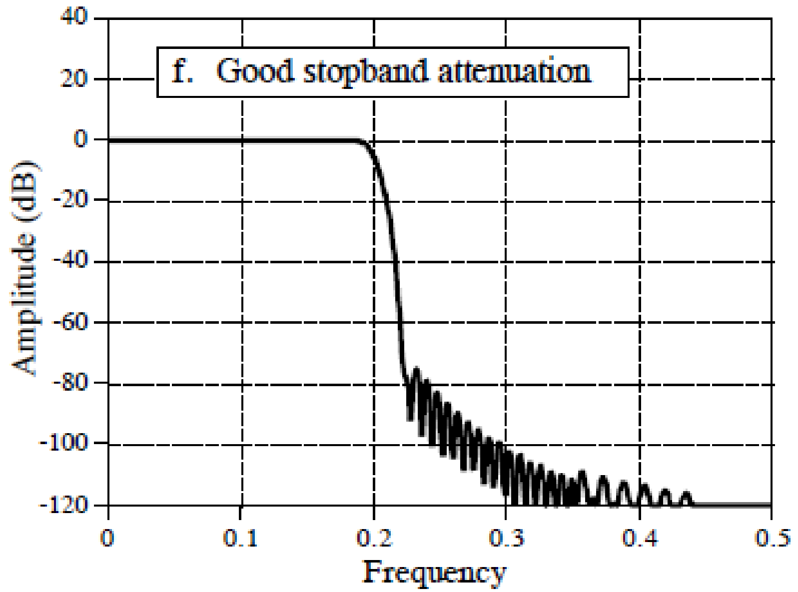
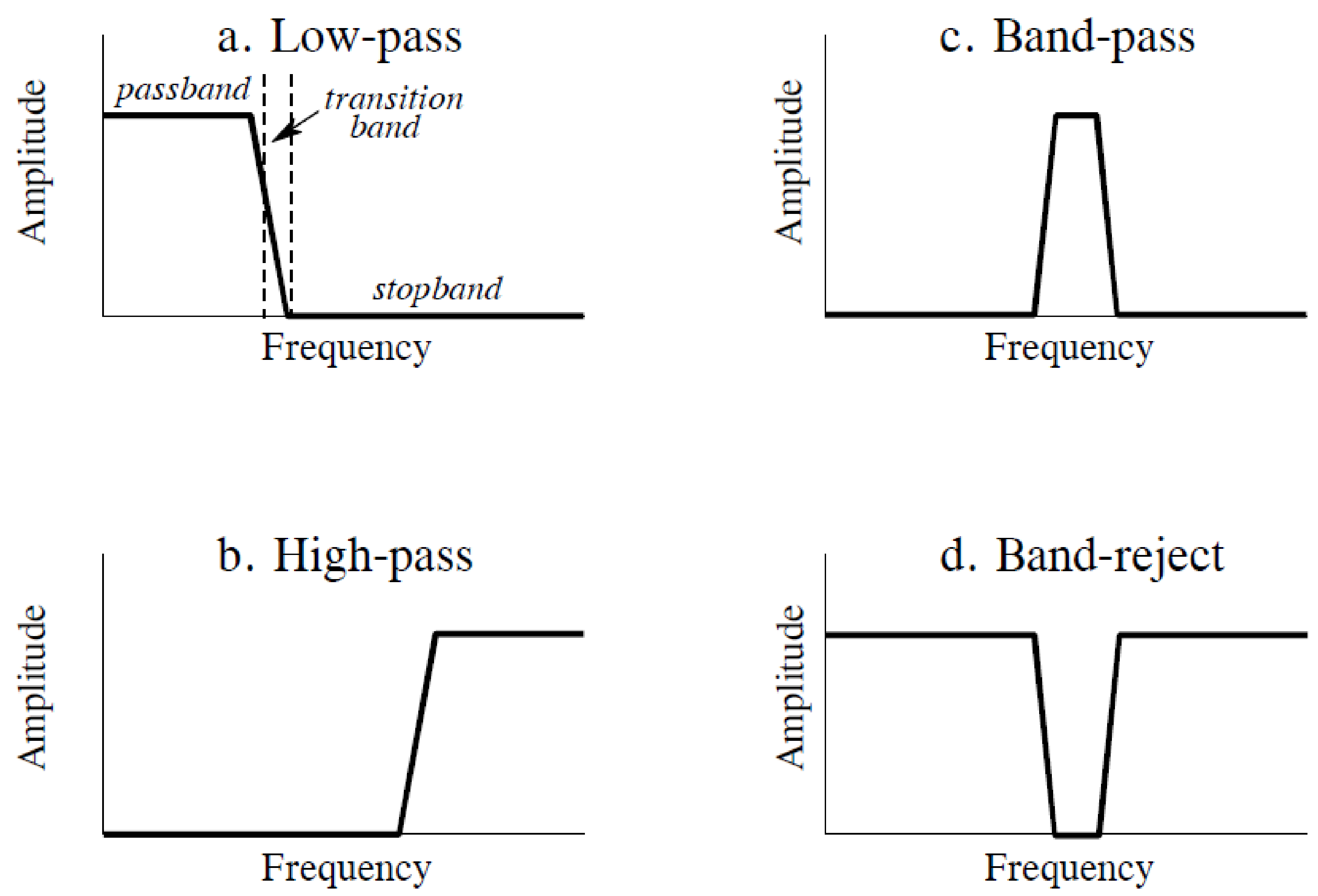
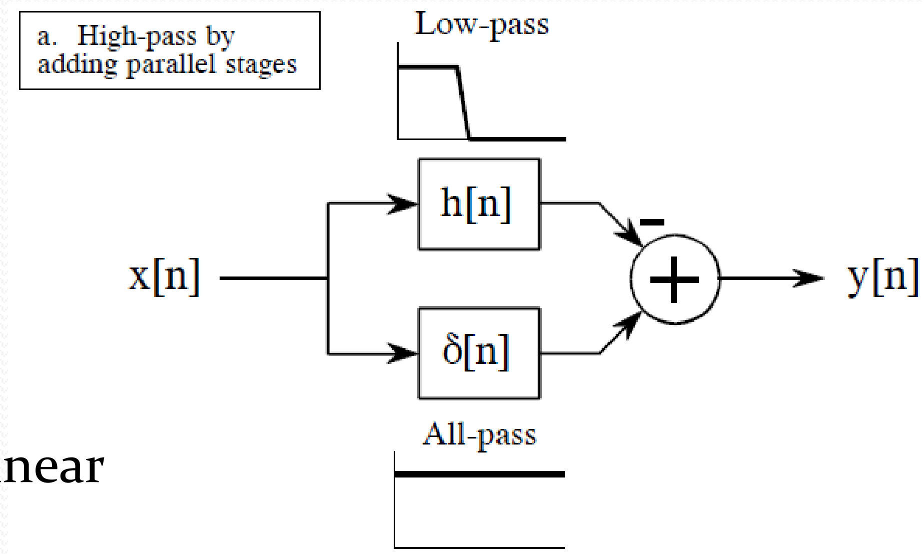


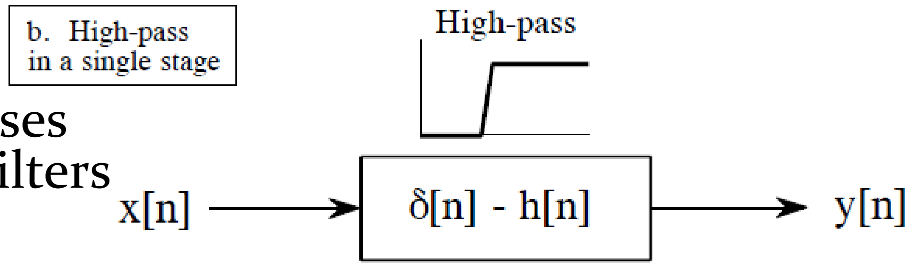
Most Common Filter Types



Converting Low Pass to High Pass

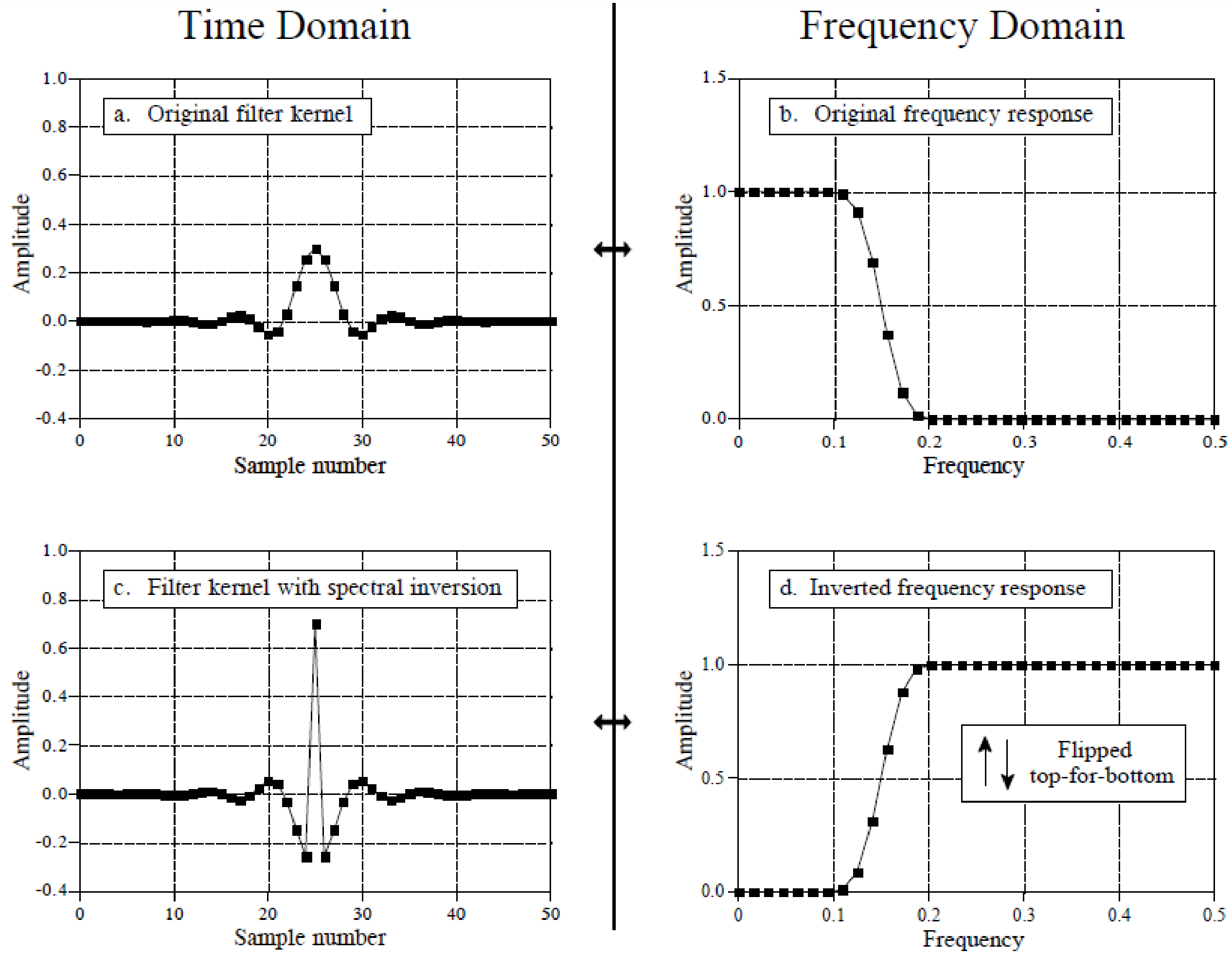
- Spectral Inversion
- Steps Done to IR:
  - Change sign of each sample
  - Add one to the center sample
- Conditions:
  - Original Filter is symmetric (Linear Phase)
  - The one must be added in the center
- Reason for Conditions:
  - To make the low frequency phases from the low pass and all pass filters the same





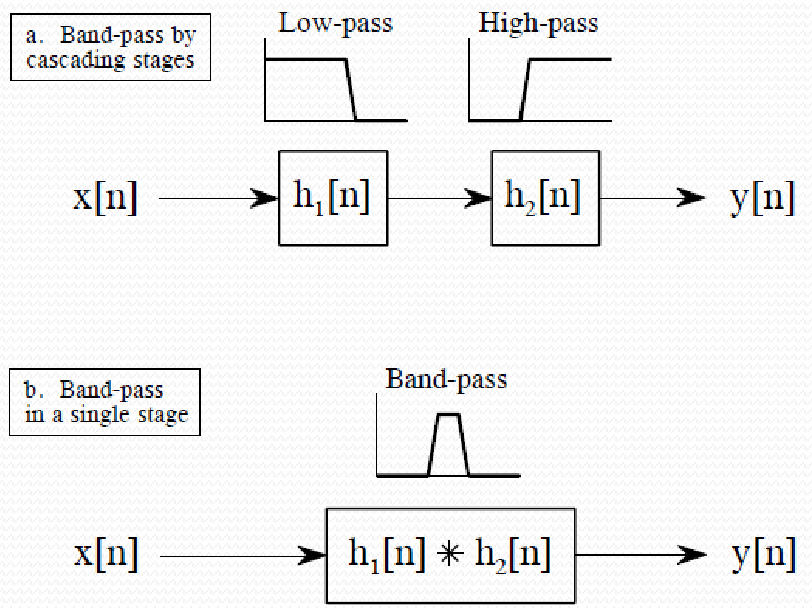
Converting Low Pass to High Pass

- Spectral Inversion



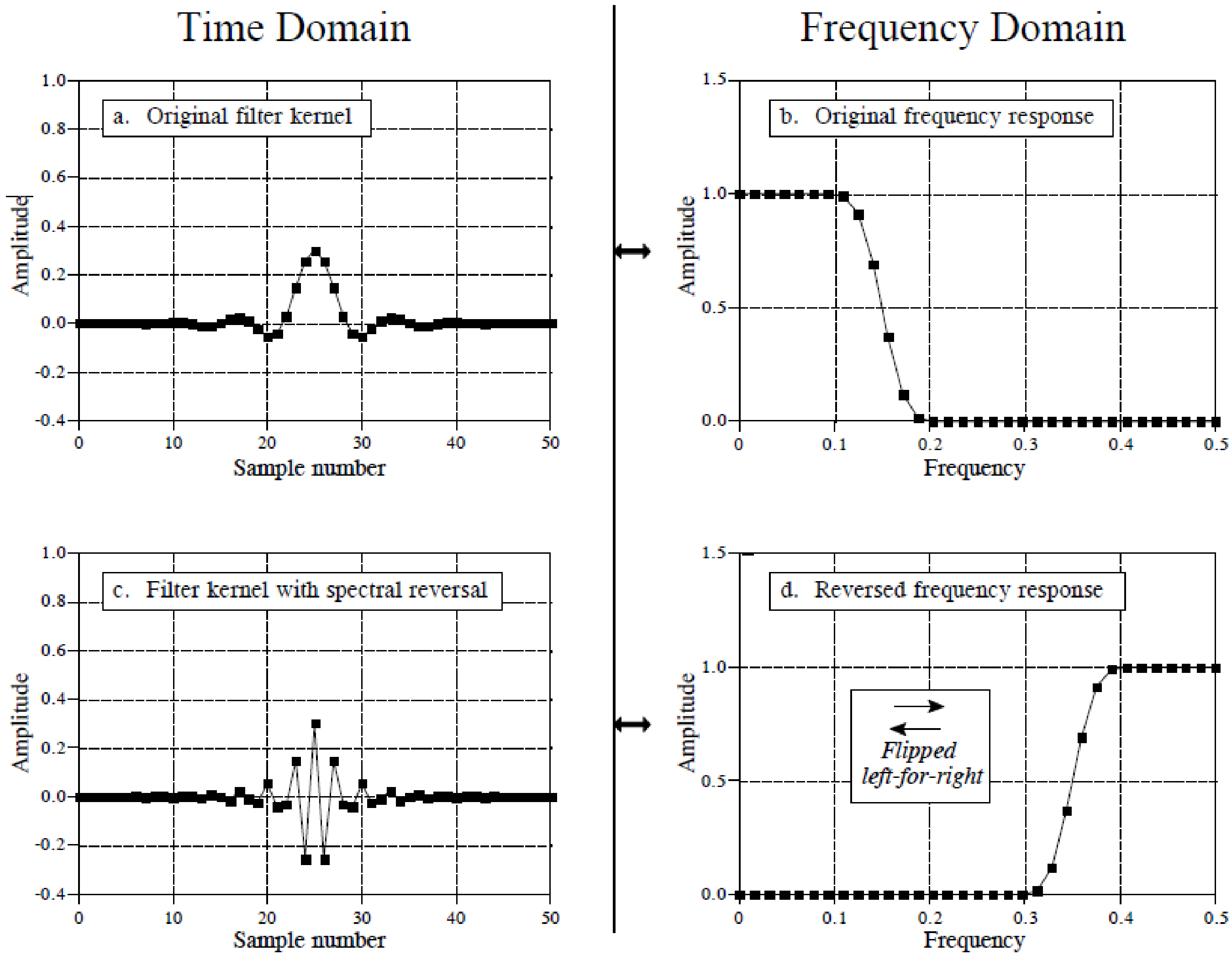
Converting Low Pass to High Pass

- Spectral Reversal
- Steps Done to IR:
  - Change sign of every other sample
- Why is it working?:
  - Multiplication with a sign of frequency of 0.5



Converting Low Pass to High Pass

- Spectral Reversal



Filter Classification

FILTER IMPLEMENTED BY:

FILTER USED FOR:	Convolution *Finite Impulse Response (FIR)*	Recursion *Infinite Impulse Response (IIR)*
Time Domain *(smoothing, DC removal)*	Moving average (Ch. 15)	Single pole (Ch. 19)
Frequency Domain *(separating frequencies)*	Windowed-sinc (Ch. 16)	Chebyshev (Ch. 20)
Custom *(Deconvolution)*	FIR custom (Ch. 17)	Iterative design (Ch. 26)